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Istanbul flower auction: The need for speed ☆

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ABSTRACT

We examine a unique auction format used in the Istanbul flower market, which could transform into either Dutch or English auction depending on bidders' bidding behaviors. By introducing a time cost that reduces the value of a perishable good as time passes, we explore how this hybrid auction format accommodates the desire for speed via an adaptive starting price. We show that the Istanbul Flower Auction outperforms both the Dutch and English auctions in terms of the auctioneer's utility. With numerical analysis, we also illustrate the Istanbul Flower Auction's superiority in terms of social welfare and auction duration. Our results highlight the critical role of auction design in improving welfare when the duration of the auction process matters.

1. Introduction

The study of auction mechanisms has long been a subject of interest in the field of economics, given their extensive application in various markets for the sale of goods and services (Klemperer, 1999; Milgrom, 1989). Auctions are particularly vital in markets where goods need to be sold quickly and efficiently, such as those for perishable items that are auctioned off in numerous lots within a predetermined period. Fresh produce auctions, which deal with the sale of perishable goods like fruits, vegetables, fish, and flowers, are critical for ensuring that these items reach consumers in a timely manner (Cassady, 1967). These markets are characterized by their need for rapid sales to facilitate transactions between a large number of sellers and buyers within very tight time frames.

Among the various auction formats, the Dutch auction has been widely adopted in markets dealing with perishable goods, praised for its speed and efficiency. In this format, the auctioneer sets a high starting price, which is progressively lowered until a bid is made, ensuring a swift sale of the item at hand. Although it is not as common, the English auction is also used to sell perishable goods (for instance at the Tokyo fish market). In an English auction, the price starts low and gradually increases, with the object awarded to the highest bidder once no further offers are made. In comparison, the Istanbul Flower Auction introduces a nuanced approach to the auction mechanism by incorporating elements of both the Dutch and English auction formats, adapting either mechanism based on the initial responses of the bidders. Specifically, given a starting price s, the Istanbul Flower Auction operates as a Dutch auction if

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no one bids at s, and it operates as an English auction (among the initial bidders) if at least one bidder bids at s. This hybrid model is innovative in its flexibility, potentially offering a superior solution to the unique challenges faced in markets where large volumes of goods need to be sold quickly and efficiently.

In this paper, we present a comparative analysis of the Istanbul Flower Auction against the backdrop of traditional Dutch and English auctions. By focusing on the perishable goods market, particularly the sale of flowers in Istanbul, we explore how different auction formats can impact the outcomes and auctioneer's and bidders' utilities. Our theoretical framework centers on the shrinking value of a perishable good, modeling the time cost as a function of the total duration of the auction. Through this lens, we study the Istanbul Flower Auction, Dutch auction, and English auction in terms of the utilities of the auctioneer and bidders in perishable goods markets. The inclusion of time discounting in our model is mainly motivated by the operational realities of the auction environment that are typical to perishable goods markets: a large volume of sales must be realized in a tight time frame. In the Istanbul Flower Auction, hundreds of lots are sold sequentially each morning, and all sales must be completed within a few hours before distribution begins. Therefore, speed of the auction is very important, and even a small time saving per sale matters. Bidders of this auction are wholesalers and retailers who face strong time constraints as they aim to purchase substantial quantities and coordinate delivery and resale activities. These institutional pressures make even short delays economically costly, providing a practical rationale for modeling time as part of bidders' payoffs.

We establish that, whenever the time cost function is weakly convex (which is a reasonably weak assumption), the Istanbul Flower Auction results in a strictly higher utility for the auctioneer compared to traditional Dutch and English auctions (Proposition 2). We also explore bidder welfare (Proposition 3), social welfare (Proposition 4) and expected auction duration (Proposition 5) across different auction formats and establish that there exist starting prices that make the Istanbul Flower Auction strictly superior to Dutch and English auctions.

We further perform numerical analyses that illustrate that for the starting price that maximizes the auctioneer's expected utility, the Istanbul Flower Auction performs better than the Dutch auction in terms of social welfare, bidder welfare, and expected auction duration. The auction's performance improves significantly when bidders are impatient, offering higher utility for the auctioneer, greater bidder payoffs, and enhanced social welfare. While its relative benefits decrease with increased market competitiveness, it still performs favorably. The analysis also shows that alternative starting prices, optimized for bidder utility, social welfare, or auction speed, still benefit the auctioneer, underscoring the mechanism's robustness.

Our study is motivated by the premise that the choice of auction format can significantly affect market outcomes, especially in sectors where time is of the essence. By examining the Istanbul Flower Auction, we seek to contribute to the broader discourse on auction theory and practice, offering insights that could inform the design and selection of auction formats in various contexts. The findings of this research may have implications for auctioneers and market organizers worldwide, suggesting that flexibility in auction design could be key to optimizing sales processes, especially in markets where a large number of items need to be sold in very tight time frames.[1]

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces our model and solves for the equilibrium of the Istanbul Flower Auction. Section 4 provides theoretical results on the payoff comparison of the Istanbul Flower Auction against the backdrop of Dutch and English auctions. Section 5 provides our numerical results. Section 6 concludes. We provide all proofs in the Appendix.

2. Literature review

There is a long strand of literature on the descending Dutch and the ascending English auctions, exploring various analogs and extensions derived from the standard models. Comprehensive overviews of these studies can be found in Klemperer (2004) and Milgrom (2004). Our contribution to the literature lies in studying a unique hybrid auction format that has been implemented in Turkish flower markets for decades.

Hybrid auctions typically allow for both ascending and descending prices throughout the auction process, with many variants in practice. For example, eBay provides a selling mechanism that gives bidders a "buy-it-now" option simultaneously with the English auction process. Mathews (2004) shows that under such circumstances, impatient sellers would gain extra revenue by setting an attractive "buy-it-now" price. Furthermore, Azevedo et al. (2020) consider a descending buyout price and attribute its advantage over standard formats to information acquisition costs. In another example, Katok and Roth (2004) study multi-unit Dutch auctions for homogeneous goods with divisible lots where the price can drop and then rise, and explain it by synergy effects between parts of the lot. More recently, Bergemann et al. (2025) propose a soft-floor auction in which bidders can accept a non-binding reserve price to enter an ascending auction, and show that when bidders experience regret from losing, such mechanisms yield higher revenue and greater efficiency than standard optimal auctions by inducing participation and enabling lower reserve prices.

Recent studies have recognized the importance of auction speed in the design of mechanisms for real-world markets. Banks et al. (2003) document the trade-off between efficiency and speed in the Federal Communications Commission spectrum auctions, where speed and revenue can be enhanced through the improved design proposed by Kwasnica et al. (2005). Andersson and Erlanson (2013) numerically show that a hybrid Vickrey-English-Dutch algorithm is faster than the Vickrey-English or Vickrey-Dutch auctions.

[1] These auctions are not only used for selling fresh produce. Wholesale car auctions, which are used by dealerships or leasing companies, can sell hundreds to thousands of vehicles in a single event (Lacetera et al., 2016). Similarly, large wine auctions can involve the sale of thousands of bottles or cases, including rare and vintage wines, over the course of a single event.

The role of speed in auctions has also been highlighted in experimental studies, where participants' impatience was linked to their enjoyment of participation (Cox et al., 1983) or intrinsic costs of time (Katok and Kwasnica, 2008).

However, none of these models formally considers "time discounting" in values due to auction duration, except for Hafalir et al. (2026), which predates our work. Hafalir et al. (2026) present a comprehensive analytical and experimental study of a hybrid auction format used in various fish markets (including Honolulu and Sydney), and demonstrate how the participants' time costs affect the auction speed and the resulting participant welfare. Unlike the fish auctions in Honolulu and Sydney, in Istanbul Flower Auctions the ascending or descending price dynamics are determined by the initial decision of bidders and cannot be reversed later. More specifically, in the Honolulu-Sydney fish auction the price can start going up after a bid in the descending Dutch phase, whereas this is not allowed in the Istanbul Flower Auction. In addition to the time-discounting modeling differences between our paper and Hafalir et al. (2026), one particular difference in the findings is that in Hafalir et al. (2026), the seller's preference for a hybrid format depends sharply on the number of bidders (the hybrid auction is preferred by the auctioneer over the Dutch auction only when the number of bidders is small), whereas in the present paper, it does not.

3. Model

We consider a single-item auction with n bidders in which the total time elapsed in the auction is important in the evaluation of the item for bidders.

3.1. Formal description of the Istanbul flower auction

The Istanbul Flower Auction proceeds as follows.[2] It begins with a starting price, s, announced by the auctioneer and bidders' decisions to *bid* or *wait* at this price. Then, the auction turns into one of the following two formats depending on the number of bidders bidding at the starting price:

(1) If no one bids at s, the auction then operates as a Dutch auction: the price starts going down until someone bids (or the auction ends with no winner when the price drops to its minimum).
(2) If at least one bidder bids at s, the auction then operates as an English auction for those initial bidders only: the price starts going up until the second last of the initial bidders leaves the auction (the rise of the price stops immediately at the starting price if there is only one bidder who bids at the starting price).[3]

3.2. Incorporating time costs

We assume that the seller has zero value for the item and the private values of the bidders are independently and identically distributed according to a twice differentiable cumulative distribution function $F(\cdot)$ over $[0,1]$ with a corresponding density function $f(\cdot)$. As the clock ticks, the time cost is modeled as a decreasing and differentiable discount factor $c(t)$ on the item value, reflecting the perishability of the item.

Specifically, if the auction ends when a bidder with value v buys at a price p after t units of time have passed, the auctioneer's utility is equal to the revenue:

$$U_A(p) = p,$$

and the winning bidder's utility is

$$U_B(p, v, t) = c(t) \cdot v - p,$$

where the time cost function $c(t)$ is a strictly decreasing function of time t.[4] More specifically, we have $c : [0,1] \to \mathbb{R}_+$,[5] where $c(0) = 1$; moreover, $c'(t) < 0$ except when $c(t) = 1$ for all $t \geq 0$. We refer to the case $c(t) = 1$ as the case of "no time cost." Note that t denotes the duration of the auction, hence when the starting price is s and the selling price is p, t is given by $t := |p - s|$. We assume that $c'(\cdot) > -1$, which ensures that the necessary first-order condition for the equilibrium (i.e., the differential Equation 1 below) has a valid solution. Bidders who lose the auction would get a utility of 0.

[2] For a more detailed description, see (Öz and Çalışkan, 2010).

[3] One could imagine alternative mechanisms that act directly on auction speed, such as allowing jump bids or restricting bidding to discrete moments (e.g., $t = 0$ or $t = 1$). However, such formats would conflict with the sequential, transparent price discovery process that defines clock-style flower auctions and would be impractical in real-world implementations. We thank the Associate Editor for suggesting to consider these alternatives.

[4] Such representation of the time cost underscores the perishable nature of the good, with more valuable items losing more value in a given time. Note that in our model, the seller does not experience the time costs directly, although a shorter auction duration also benefits the seller through better preserving the value of the good. In practice, perishable good auctions do not employ reserve prices, as delaying the sale until later is not an option. Indeed, the good would lose its freshness (if not its entire value) if it is not sold "here and now."

[5] Note that the total duration of the auction is at most 1.

3.3. Equilibrium behavior

We begin our analysis with the bidder behavior. Bidders simultaneously decide on whether to bid at the starting price. We assume that these decisions are made instantly, without the implied time cost. Moreover, in equilibrium, the decision to bid also reflects each bidder's preference for the ascending or descending auction dynamics in a consistent way such that she will not join the other dynamics.

Consider a starting price $s \in [0,1]$ and bidders implementing symmetric bidding strategies that are increasing in value. They need to decide on (i) whether to bid or wait at the starting price and (ii) at what price to bid or leave when they are active bidders in the subsequent Dutch or English auction phases.

We first discuss the bidder decision to bid or wait at the starting price. Intuitively, it is always a dominated strategy for a bidder with item value $v < s$ to bid at the opening, because she can never acquire a positive payoff in the subsequent English auction phase. Therefore, as we restrict our attention to the bidding strategies that are monotonically increasing in value, we focus on the case featuring a cutoff $p(s) \in [s,1]$ such that the bidder only bids at the starting price if her private value is greater than or equal to $p(s)$, and waits otherwise. In the remainder of the analysis, we use the common "first-order approach" in auction theory literature, assuming the existence of a symmetric equilibrium with increasing bidding strategies. This equilibrium is characterized by the first-order conditions for the bidder optimization problems.

We now define several auxiliary functions that will be useful later. Let us denote the distribution of the highest of $n-1$ random variables identically and independently distributed according to $F(\cdot)$ by a cumulative distribution function $G(\cdot)$ with a corresponding density function $g(\cdot)$, i.e., $G(v) = F^{n-1}(v)$. Let us also denote the joint density function of the highest value being v and the second highest value being x among n random variables identically and independently distributed according to $F(\cdot)$ by $h(v,x)$, i.e., $h(v,x) = n(n-1)f(v)f(x)F^{n-2}(x)$.

In the subsequent Dutch auction phase, given the starting price s, let us denote the symmetric equilibrium bidding function by $b(v,s)$. Since this Dutch phase starts from price s and not from 1, it could be possible that there is a cluster of bids at s: i.e., we may have $b(v,s) = s$ for all $v \in [\lambda(s), p(s)]$ for some $\lambda(s) \in (0, p(s))$. We assume ties break evenly for those bidders potentially choosing to cluster their bids at s. Lemma 1 shows that, in equilibrium, there will be no cluster bids at s, and the value cutoff $p(s)$ is determined by the starting price s via the Dutch bidding function when $p(s) < 1$.

Lemma 1. *In a symmetric equilibrium with increasing bidding strategies, the probability that a tie occurs in the Dutch auction phase is zero, i.e., there are no cluster bids at the starting price s. Moreover, the unique solution to the cutoff value $p(s)$ is given by $b(p(s),s) = s$ when $p(s) < 1$. When $p(s) = 1$, we have $b(p(s),s) \le s$.*

We would like to note that under our assumptions (symmetric equilibrium with increasing bidding strategies), Lemma 1 establishes that the Dutch auction equilibrium bidding strategies are unique.

A necessary condition for $b(v,s)$ to be a symmetric equilibrium strategy is that the bidder maximizes her payoff at her own value v instead of any alternative z. For $v, z \le p(s)$, it is the Dutch auction problem (in the presence of time costs):

$$v = \arg\max_z G(z)[c(s - b(z,s))v - b(z,s)]$$

which leads to the following ordinary differential equation derived from the first-order condition:

$$\frac{\partial b(v,s)}{\partial v} = \frac{g(v)}{G(v)} \frac{c(s - b(v,s))v - b(v,s)}{1 + c'(s - b(v,s))v}. \tag{1}$$

Indeed, the Dutch phase bidding strategy $b(v,s)$ is defined by the differential Eq. (1)[6] and the initial condition $b(0,s) = 0$ for all s.

Now, consider a bidder with value v who bids at the starting price to initiate the English auction phase, i.e., when $v \ge p(s)$. She will remain in the auction and compete with other bidders (if any) until the price reaches the level at which her utility drops to zero. Let us denote this English phase leaving strategy in the symmetric equilibrium by $m(v,s)$. It is defined by the following equation:

$$c(m(v,s) - s)v - m(v,s) = 0. \tag{2}$$

From the basic assumptions of the time cost function, $m(v,s)$ is a monotonic increasing function in the item value v.[7] The expected utility for a bidder who bids in the English phase is given by:[8]

$$EU_B^{FE}(v,s) = (v - s)G(p(s)) + \int_{p(s)}^{v} [c(m(x,s) - s)v - m(x,s)]\, dG(x),$$

where the first term is her expected utility from winning the item at the starting price when no other bidder is competing with her, and the second term is her expected utility from winning the item at the highest price among other competitors.

As such, when an Istanbul Flower Auction starts at s, the ex-ante expected utility for a bidder is given by

$$EU_B^F(s) = \int_0^{p(s)} [c(s - b(v,s))v - b(v,s)]G(v)\, dF(v) + \int_{p(s)}^{1} \left((v - s)G(p(s)) + \int_{p(s)}^{v} [c(m(x,s) - s)v - m(x,s)]\, dG(x) \right) dF(v). \tag{3}$$

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[6] Since $c'(\cdot) > -1$, we have $1 + c'(s - b(v,s))v > 0$.

[7] Taking the derivative with respect to v on both sides of the equation, we have $\frac{dm(v,s)}{dv} = \frac{c(m(v,s),s)}{1 - c' m(v,s) - s} > 0$.

[8] The superscript FE corresponds to the Istanbul Flower Auction in the English auction phase. Similarly, FD: Istanbul Flower Auction in the Dutch auction phase, D: Dutch auction, E: English auction, F: Istanbul Flower Auction.

The resulting utility for the auctioneer is given by

$$EU_A^F(s) = \int_0^{p(s)} b(v,s)\,dF^n(v) + \int_{p(s)}^1 \left(\int_0^{p(s)} sh(v,x)dx + \int_{p(s)}^{v} m(x,s)h(v,x)\,dx \right) dv. \tag{4}$$

Now, the optimization problem for the auctioneer is to select the starting price s^* that maximizes her utility. That is,

$$s^* = \arg\max_s EU_A^F(s)$$

We can argue that the equilibrium we consider for the Istanbul Flower Auction is "value-efficient." The reasons are that: (i) the Dutch phase bidding function is assumed to be increasing in the bidders' item values, and (ii) the English phase leaving function is increasing in the bidders' item values. The following remark notes the value-efficiency of the Istanbul Flower Auction.

Remark 1. In equilibrium, the Istanbul Flower Auction is value-efficient, in the sense that the item is allocated to the bidder with the highest item value.

Note that due to the presence of time costs, value-efficiency does not imply the social welfare maximization, as the latter includes both the auctioneer's and the bidders' utilities. We investigate the social welfare and the duration of the Istanbul Flower Auction in addition to the auctioneer and bidder utilities. The corresponding definitions are given below.

The expected social welfare of the auction is defined as the aggregated expected utility for all market participants, $EU_S^F(s) = EU_A^F(s) + n \cdot EU_B^F(s)$. We can calculate it as

$$EU_S^F(s) = \int_0^{p(s)} c(s-b(v,s))v\,dF^n(v) + \int_{p(s)}^1 \left(\int_0^{p(s)} vh(v,x)\,dx + \int_{p(s)}^{v} c(m(x,s)-s)vh(v,x)\,dx \right) dv. \tag{5}$$

The expected duration of the auction is given by

$$ED^F(s) = \int_0^{p(s)} [s-b(v,s)]\,dF^n(v) + \int_{p(s)}^1 \left(\int_0^{p(s)} 0 \cdot h(v,x)\,dx + \int_{p(s)}^{v} [m(x,s)-s]h(v,x)\,dx \right) dv. \tag{6}$$

3.4. Dutch and English auctions

The Istanbul Flower Auction inherently contains elements of both English and Dutch auctions and can be explicitly converted to either format by setting the initial price at $s = 0$ or $s = 1$.

When $s = 1$, as in the Dutch auction, the price continuously descends from 1 until the first bidder stops the clock and wins the item. In this case, the ex-ante expected utilities for the auctioneer and the bidders, the social welfare and the expected duration are given by

$$\begin{aligned}
EU_A^D &= EU_A^F(1) = \int_0^1 b(v,1)\,dF^n(v),\\
EU_B^D &= EU_B^F(1) = \int_0^1 [c(1-b(v,1))v - b(v,1)]G(v)\,dv,\\
EU_S^D &= EU_S^F(1) = \int_0^1 c(1-b(v,1))v\,dF^n(v),\\
ED^D &= ED^F(1) = \int_0^1 [1-b(v,1)]\,dF^n(v).
\end{aligned}$$

The English auction is also incorporated into the Istanbul Flower Auction framework by setting the starting price at $s = 0$. In this case, the aforementioned four auction characteristics are given by

$$\begin{aligned}
EU_A^E &= EU_A^F(0) = \int_0^1 \int_0^v m(x,0)h(v,x)\,dx\,dv,\\
EU_B^E &= EU_B^F(0) = \int_0^1 \int_0^v [c(m(x,0))v - m(v,0)]\,dG(x)\,dF(v),\\
EU_S^E &= EU_S^F(0) = \int_0^1 \int_0^v c(m(x,0))vh(v,x)\,dx\,dv,\\
ED^E &= ED^F(0) = \int_0^1 \int_0^v m(x,0)h(v,x)\,dx\,dv.
\end{aligned}$$

Hence, the Istanbul Flower Auction can be viewed as a hybrid mechanism that is conducted as either an English or a Dutch auction. The initial price calibrates the occurrence of each format.

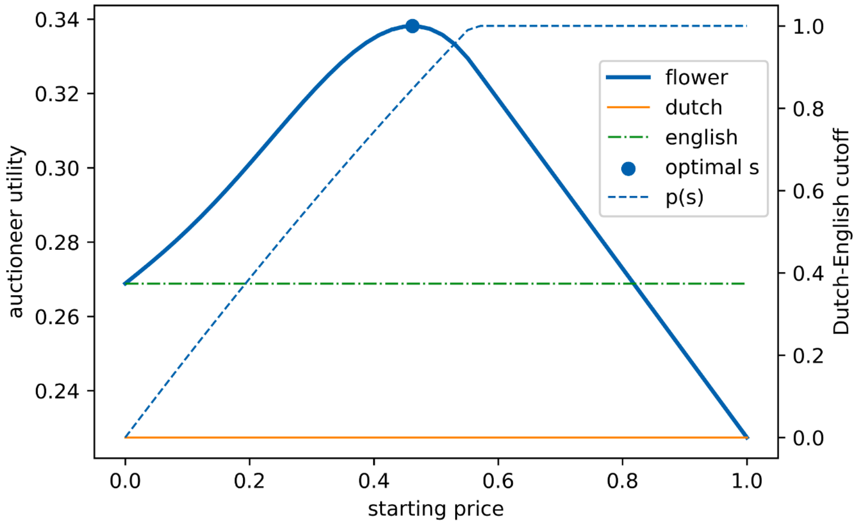


Fig. 1. Expected utility of the auctioneer in two-bidder auctions with uniform value distribution and linear cost of time $c(t) = 1 - 0.5t$.

3.5. An illustrative example

Consider the Istanbul Flower Auction with two bidders where the time cost is $c(t) = 1 - 0.5t$. The private value of each bidder is independently and uniformly distributed in $[0, 1]$. The auctioneer sets the starting price s.

The cutoff value $p(s)$ that sorts the bidders into those who bid and those who wait at the starting price is determined by equating the expected utility of the cutoff bidder in the Dutch phase and the English phase. If the auction starts as a Dutch auction, the Dutch bidding strategy $b(v, s)$ is determined by the differential equation below with the initial condition $b(0, s) = 0$:

$$\frac{\partial b(v,s)}{\partial v} = \frac{(1-0.5s)v - (1-0.5v)b(v,s)}{v(1-0.5v)}.$$

The expected utility of the cutoff bidder with value p in the Dutch phase is given by

$$EU_B^{FD}(p,s) = ([1 - 0.5(s - b(p,s)]p - b(p,s)) \cdot p.$$

Otherwise, the auction starts as an English auction, and the English leaving decision $m(v, s)$ can be solved from

$$[1 - 0.5(m(v,s) - s)]v - m(v,s) = 0,$$

which gives

$$m(v,s) = \frac{1+0.5s}{1+0.5v}v.$$

The expected utility of the cutoff bidder is given by

$$EU_B^{FE}(p,s) = (p - s) \cdot p.$$

We numerically solve p from $EU_B^{FD}(p,s) = EU_B^{FE}(p,s)$ and observe that $p(s)$ increases from 0 to 1 in s for $s \in (0, 0.557)$ and remains constant at 1 for $s \in (0.557, 1)$.

By substituting in the functions $p(s)$, $b(v, s)$ and $m(v, s)$ calculated above, we can numerically find that the auctioneer's utility is maximized at the starting price $s^* = 0.462$. The cutoff value at the optimal starting price is $p(s^*) = 0.847$, which means that the bidders with item values $v \in [0, 0.847]$ will wait for the Dutch phase, while the bidders with item values $v \in (0.847, 1]$ will bid to start the English phase at the beginning of the auction.

Fig. 1 compares the auctioneer's payoff in the Istanbul Flower Auction with the Dutch auction ($s = 1$) and the English auction ($s = 0$). The expected utility for the auctioneer at the optimal starting price is 0.338, which is the peak of the utility curve. It outperforms the Dutch auction expected utility of 0.227 by 49%, and outperforms the English auction expected utility of 0.269 by 26%. Yet notice that in this example the Istanbul Flower Auction can be worse than the English auction for the auctioneer if she sets a sufficiently high starting price. This illustrates the importance of setting the starting price optimally.

Under the optimal choice of the starting price, we can further calculate that the expected duration of the Istanbul Flower Auction is 0.138, which is 82% faster than the Dutch auction. Bidders are also expected to earn 53% more than in the Dutch auction, which ultimately increases the social welfare by 51%.

4. Auction characteristics analysis

4.1. Superiority of the optimal Istanbul flower auction for the auctioneer

When there is no time cost, the classic payoff equivalence result (Myerson, 1981; Riley and Samuelson, 1981) can be obtained from Remark 1. We document it in the following proposition.

Proposition 1. *The Istanbul Flower Auction satisfies the Payoff Equivalence Principle when bidders have no time costs, i.e.,* $EU_A^F(s) = \int_0^1 \left[v - \frac{1-F(v)}{f(v)}\right] dF^n(v)$ *when* $c(t) = 1$.

However, when the bidders incur time costs, the payoff equivalence principle no longer holds. In this more interesting case, we provide comparative static analyses based on the local properties of the auctioneer utility function in our time-costly setting.

The first-order derivative of the expected utility for the auctioneer in Istanbul Flower Auction is given by

$$\begin{aligned}\frac{dEU_A^F(s)}{ds} =& -\frac{dp(s)}{ds}[s - b(p,s)]nF^{n-1}(p(s))f(p(s)) \\ &- \frac{dp(s)}{ds}[m(p(s),s) - s]n(n-1)[1-F(p(s))]F^{n-2}(p(s))f(p(s)) \\ &+ n[1-F(p(s))]F^{n-1}(p(s)) \\ &+ \int_{p(s)}^1 \int_{p(s)}^v \frac{\partial m(x,s)}{\partial s} h(v,x)\, dx\, dv \\ &+ \int_0^{p(s)} \frac{\partial b(v,s)}{\partial s}\, dF^n(v).\end{aligned} \tag{7}$$

Recall that $EU_A^D = EU_A^F(1)$. Hence, to demonstrate that the Istanbul Flower Auction is preferred by the auctioneer to the Dutch auction, it is sufficient to show that the expected utility of the auctioneer is decreasing at $s = 1$ (or the derivative of the expected utility at $s = 1$ is negative), since then it must reach a higher utility for some interior s. Similarly, since $EU_A^E = EU_A^F(0)$, the Istanbul Flower Auction is better than the English auction if the expected utility for the auctioneer is increasing at $s = 0$ (or the derivative of the expected utility at $s = 0$ is positive). That is, the payoff comparison between the auction formats follows naturally from establishing the sign of the derivative of the auctioneer's expected utility with respect to the starting price at the corresponding boundary values.

It is evident from equation (7) above that the properties of the terms $\frac{\partial b(v,s)}{\partial s}$, $\frac{\partial m(v,s)}{\partial s}$ and $\frac{dp(s)}{ds}$ are crucial in determining the sign of the derivative.

We first note that when varying the starting price s, the variation in the Dutch phase bidder strategy $\frac{\partial b(v,s)}{\partial s}$ has an upper bound, and the variation in the English phase bidder strategy $\frac{\partial m(v,s)}{\partial s}$ is bounded:

Lemma 2. $\frac{\partial b(v,s)}{\partial s} < 1$.

Lemma 3. $0 \le \frac{\partial m(v,s)}{\partial s} < 1$*, and the equality only holds when there is no time cost, i.e.* $c(\cdot) \equiv 1$.

Intuitively, the variation of the starting price is only partially reflected in the variation of the Dutch-phase or English-phase bidding strategy.

Next, we describe how the bidder Dutch-English cutoff value $p(s)$ in the Istanbul Flower Auction changes with the starting price s set by the auctioneer:

Lemma 4. *There exists a starting price* $\tilde{s} \in (0,1)$ *such that*

(a) $p(s) < 1$ *and* $\frac{dp(s)}{ds} > 0$ *when* $s \in [0, \tilde{s})$*;*
(b) $p(s) = 1$ *and* $\frac{dp(s)}{ds} = 0$ *when* $s \in [\tilde{s}, 1]$.

The bidder cutoff value $p(s)$ is monotonically increasing in the starting price as long as the starting price is below a certain value $\tilde{s}$; in this case, the Istanbul Flower Auction can indeed proceed into either the Dutch or the English phase. If the starting price is set at or above $\tilde{s}$, the Dutch-English bidder cutoff value is fixed at 1. In that case, all bidders will wait for the Dutch phase, and the Istanbul Flower Auction simply becomes a Dutch auction with a starting price s.

This lemma also helps us conceptualize the "non-triviality" of the Istanbul Flower Auction dynamics from the perspective of the auctioneer who is choosing the starting price among all possible $s \in [0,1]$: (i) the auction is *non-trivial* if $s \in (0, \tilde{s})$, in which case different bidders may differ in their decisions to wait for the Dutch phase or to bid in the English phase, depending on their value; while (ii) the auction is *trivial* if either $s = 0$, in which case all bidders bid in the English phase, or if $s \in [\tilde{s}, 1]$, in which case all bidders wait for the Dutch phase.

So far, we have only established the upper bound of 1 for $\frac{\partial b(v,s)}{\partial s}$, which is insufficient for further analysis. Further insights into how the Dutch phase bidding function reacts to changes in the starting price can be obtained by imposing a mild assumption on the time cost function:

Assumption 1. The time cost function $c(t)$ is convex in its domain, i.e., $c''(t) \ge 0$ for all $t \in [0,1]$.

Note that this assumption is satisfied for many standard time-cost functions used in the literature. Specifically, the linear cost $c_B(t) = 1 - \mu t$, the exponential cost $c_B(t) = e^{-\mu t}$, and the hyperbolic cost $c_B(t) = \frac{1}{1+\mu t}$ all satisfy this assumption when the parameter $\mu \in [0, 1)$.

The convexity of the time cost function ensures a negative sign of $\frac{\partial b(v,s)}{\partial s}$, which is given by the lemma below:

Lemma 5. *In equilibrium, the bidder's Dutch phase bidding function $b(v, s)$ in the Istanbul Flower Auction is strictly decreasing in the starting price s when there is a time cost and Assumption 1 is satisfied. That is, $\forall v \in (0, p(s))$, $\frac{\partial b(v,s)}{\partial s} < 0$ when $c'(t) < 0, c''(t) \geq 0$. In addition, $\frac{\partial b(0,s)}{\partial s} \equiv 0$.*

With the assumption and the lemmas introduced above, the payoff comparison between the Istanbul Flower Auction and the Dutch or English auctions can be established. We now state our primary analytical finding on the general advantage of the Istanbul Flower Auction.

Proposition 2. *When there is a time cost and Assumption 1 is satisfied, the optimal Istanbul Flower Auction is strictly better than both the Dutch auction and the English auction for the auctioneer, and the resulting auction dynamics is non-trivial. That is, if $c'(t) < 0$ and $c''(t) \geq 0$, then $s^* \in (0, \tilde{s})$, and $EU_A^F(s^*) > EU_A^D$, $EU_A^F(s^*) > EU_A^E$.*

4.2. Distinct advantages of the Istanbul flower auction from alternative perspectives

Throughout the paper, the optimal Istanbul Flower Auction is derived from the perspective of the auctioneer; and we use the term "optimal" and "auctioneer-optimal" interchangeably. However, the most preferred starting prices for other parties may be different from the one chosen by the auctioneer. The three propositions below confirm the existence of a non-trivial Istanbul Flower Auction that strictly outperform both the Dutch auction and the English auction when the focus of optimization shifts to the bidder utility, the social welfare or the speed of the auction, respectively. To emphasize the difference from the auctioneer's optimal starting price s^*, the Istanbul Flower Auction starting prices for these improvements are denoted by $\hat{s}$.

Proposition 3. *When there is a time cost and Assumption 1 is satisfied, there exists a starting price $\hat{s}_B$ such that the corresponding Istanbul Flower Auction is strictly better than both the Dutch auction and the English auction for the bidders, and the resulting auction dynamics is non-trivial. That is, $\exists \hat{s}_B \in (0, \tilde{s})$ such that $EU_B^F(\hat{s}_B) > \max\{EU_B^D, EU_B^E\}$.*

Proposition 4. *When there is a time cost and Assumption 1 is satisfied, there exists a starting price $\hat{s}_S$ such that the corresponding Istanbul Flower Auction is strictly better than both the Dutch auction and the English auction in terms of the social welfare, and the resulting auction dynamics is non-trivial. That is, $\exists \hat{s}_S \in (0, \tilde{s})$ such that $EU_S^F(\hat{s}_S) > \max\{EU_S^D, EU_S^E\}$.*

Proposition 5. *When there is a time cost and Assumption 1 is satisfied, there exists a starting price $\hat{s}_D$ such that the corresponding Istanbul Flower Auction is strictly better than both the Dutch auction and the English auction in terms of the auction duration, and the resulting auction dynamics is non-trivial. That is, $\exists \hat{s}_D \in (0, \tilde{s})$ such that $ED^F(\hat{s}_D) < \min\{ED^D, ED^E\}$.*

The proofs of the above propositions follow the same logic that we used to prove the superiority of the optimal Istanbul Flower Auction for the auctioneer by demonstrating that (1) the target function is increasing at the trivial English-equivalent starting price $s = 0$, and (2) the target function is decreasing in the trivial Dutch-equivalent starting price interval $s \in (\tilde{s}, 1)$.

The proof methods of the above propositions also establish a "win-win" scenario in the following sense. The Istanbul Flower Auction with a starting price slightly less than 1 would result in a higher auctioneer utility and bidder (ex-ante) utility, as well as a lower expected duration, as compared to the Dutch auction. Similarly, the Istanbul Flower Auction with a starting price slightly greater than zero would result in a higher auctioneer utility and bidder (ex-ante) utility, as well as a lower expected duration, as compared to the English auction.

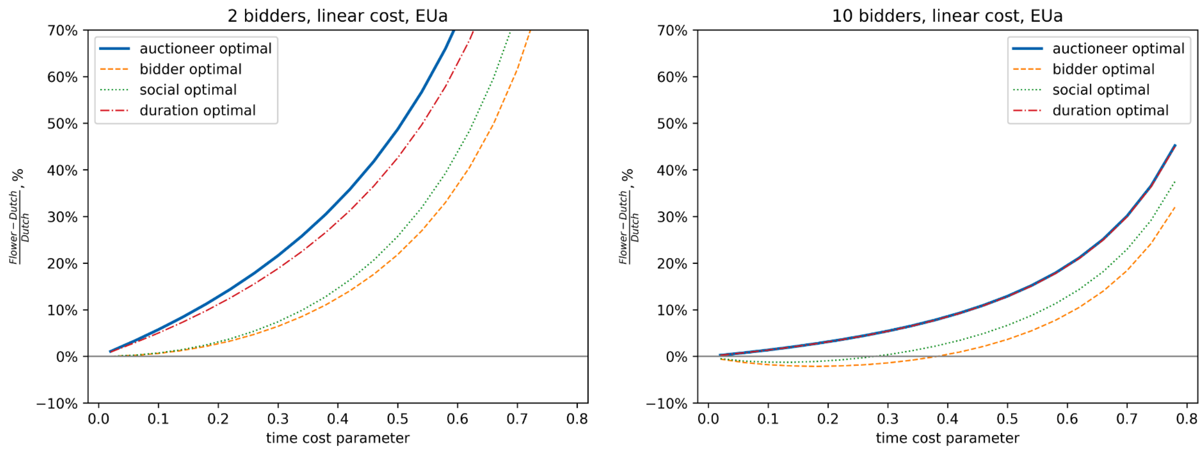
Deriving analytical results at the auctioneer-optimal starting price s^* for these auction characteristics proves challenging. As a substitute, we present supportive computational results in the next section where we also decompose the payoff differences between the Istanbul Flower Auction and the Dutch auction into duration and price effects. Our numerical findings illustrate that, under a wide range of parameter choices, the auctioneer-optimal Istanbul Flower Auction is also a better choice than the Dutch auction for the bidders and the social planner, and it is faster.

5. Numerical analyses

In this section, we provide further evidence for the advantages of the Istanbul Flower Auction using numerical simulations. Although our analyses confirm the advantages of the Istanbul Flower Auction over both standard auction formats, here we focus on the comparison with the Dutch auction, while the comparison with the English auction is relegated to Online Appendices A and B. This is mainly due to the fact that the Dutch auction is the most commonly used auction format for perishable goods, while the use of the English auction is much less frequently observed.[9]

In Section 5.1 below, we consider how the benefits of the Istanbul Flower Auction for the auctioneer, the bidders and the social welfare vary with bidder time costs, market competitiveness, and choice of starting price. In Section 5.2 we analyze the channels that make the Istanbul Flower Auction format more beneficial for the auctioneer and the bidders.

[9] There is a long history of the dominant use of the Dutch auction in the sale of fresh flowers. In fact, the name of the Dutch auction comes from the famous tulip auction in 17th-century Holland. However, for other perishable goods, English (ascending) auctions are also observed in some contexts, for example, in the Tokyo fish market.



(a) 2 bidders (b) 10 bidders

Fig. 2. Auctioneer utility gain under Istanbul Flower Auction relative to Dutch auction with varying time cost, percent.

5.1. Comparative statics and performance robustness under alternative criteria

We now explore the answers to the following questions:

- How will the auctioneer's benefit in the optimal Istanbul Flower Auction vary with the changes in the time cost parameter, and with market competitiveness (i.e., the number of bidders)?
- Will the auctioneer-optimal Istanbul Flower Auction enhance other benefits such as bidder utility, social welfare, and auction speed?
- If the starting price is set to enhance other aspects of the Istanbul Flower Auction, rather than prioritize auctioneer's benefits, will the auction still serve the auctioneer's interests?

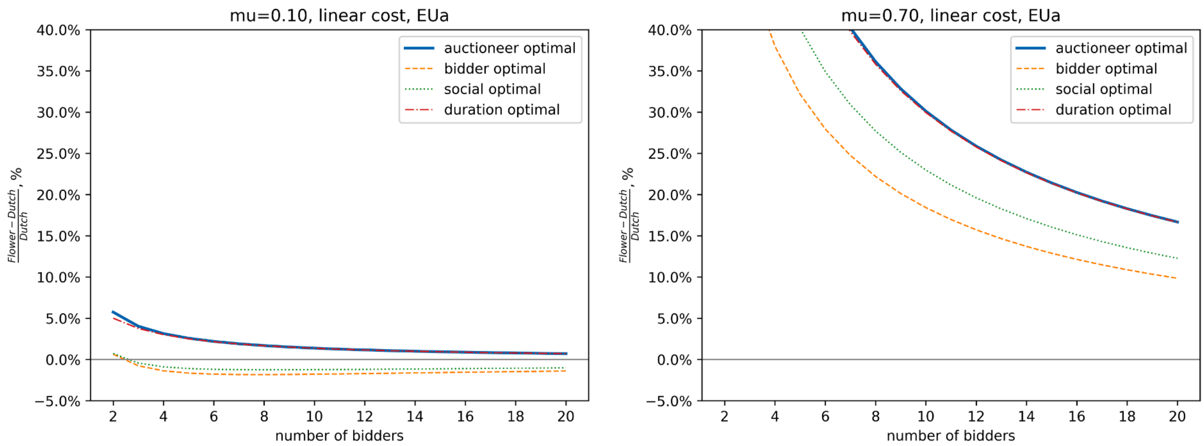
As noted previously, typical functional forms for time costs, such as linear, exponential, and hyperbolic costs, all satisfy Assumption 1. For simplicity, we consider the linear time cost function $c(t) = 1 - \mu t$, with the parameter $\mu \geq 0$ indicating the impatience of bidders, and assume a uniform distribution of item values $F(v) = v$ over $[0, 1]$.

The results of numerical simulations illustrating the advantage of the Istanbul Flower Auction relative to the Dutch auction in terms of auctioneer utility, calculated as $\frac{EU^F - EU^D}{EU^D}$, are displayed in Figs. 2 and 3. In Fig. 2, we illustrate the effect of impatience of the bidders by varying the time cost parameter from completely patient ($\mu = 0$) to highly impatient ($\mu = 0.8$) in auctions with either a small number of bidders ($n = 2$) or a relatively large number of bidders ($n = 10$). In Fig. 3, we illustrate the effect of the market competitiveness by varying the number of bidders in auctions from 2 to 20 with either relatively patient bidders ($\mu = 0.1$) or highly impatient bidders ($\mu = 0.7$).

Auctioneer utility at the optimal starting price. In Figs. 2 and 3, first consider the blue lines that represent the performance of the Istanbul Flower Auction with the auctioneer-optimal starting price. The relative advantage of the optimal Istanbul Flower Auction over the Dutch auction increases when bidders are more impatient (see Fig. 2), while it decreases when the market becomes more competitive (see Fig. 3). When bidders are relatively patient, the optimal Istanbul Flower Auction outperforms the Dutch auction by less than 10% (see panel (a) in Fig. 3), whereas when bidders are highly impatient, it outperforms the Dutch auction by more than 15% even in highly competitive markets with more than 10 bidders (see panel (b) in Fig. 3).

Auctioneer utility at alternative starting prices. Now consider the other lines in Figs. 2 and 3, which display the auctioneer's relative benefit of the Istanbul Flower Auction over the Dutch auction when the starting price is set to maximize the expected utility for the bidders (orange dashed lines), the expected social welfare (green dotted lines) or to minimize the expected duration of the auction (red dash-dotted lines). We observe that the auctioneer can still significantly benefit from the Istanbul Flower Auction when bidders are very impatient. In contrast, the auctioneer can be slightly worse off (no more than by 3%) in the Istanbul Flower Auction than in the Dutch auction when bidders are relatively patient. This suggests that the significant advantage of the Istanbul Flower Auction over the Dutch auction is robust to alternative optimization targets. The auctioneer incurs very little loss and typically benefits a lot from the Istanbul Flower Auction compared to the Dutch auction when she prioritizes the satisfaction of bidders, the social welfare, or the speedy sale.

Welfare and speed performance at the auctioneer-optimal starting price. Table 1 shows that the bidders, the social planner, and the time-saver (duration minimizer) all benefit from the auctioneer-optimal Istanbul Flower Auction compared to the Dutch auction. The



(a) Low cost (b) High cost

Fig. 3. Auctioneer utility gain under Istanbul Flower Auction relative to Dutch auction with varying number of bidders, percent.

Table 1
Optimal Istanbul Flower Auction performance relative to Dutch.

	2 bidders		10 bidders	
	$\mu = 0.1$	$\mu = 0.7$	$\mu = 0.1$	$\mu = 0.7$
Auctioneer payoff Flower/Dutch, %	105.7%	207.7%	101.4%	130.1%
Buyer payoff Flower/Dutch, %	106.1%	222.7%	101.7%	140.4%
Social welfare Flower/Dutch, %	105.9%	214.2%	101.4%	131.0%
Auction duration Flower/Dutch, %	18.9%	17.2%	28.8%	16.6%

relative advantage of the Istanbul Flower Auction is always greater with more impatient bidders. As the number of bidders increases, the improvements made by the Istanbul Flower Auction in terms of auctioneer utility, buyer payoff and social welfare become smaller. Moreover, the speed advantage of the Istanbul Flower Auction relative to the Dutch auction continues to be very high irrespective of market competitiveness with highly impatient bidders, while this advantage slightly decreases in more competitive markets with relatively patient bidders.

5.2. *Time-saving and bidding-strategy effects*

We now take a closer look at the sources of the advantage of the Istanbul Flower Auction over the Dutch auction. As both auction mechanisms have the item allocated to the highest-valued bidder, the allocation is not affected by the auction format. The outcomes of the Istanbul Flower Auction differ from the Dutch auction in two aspects — duration and price. Accordingly, we decompose the performance difference between the Istanbul Flower Auction and the Dutch auction into the time-saving (duration) effect and the bidding-strategy (price) effects.[10] Given the model specifications, the auctioneer's direct benefit from the Istanbul Flower Auction comes through the price channel, as she does not possess a time cost. The duration and price effects are more intertwined when it comes to the bidder's utility. As we detail below, we numerically show that on average, bidders benefit from a shorter duration of the Istanbul Flower Auction, but pay a higher price as compared to the Dutch auction. The auctioneer then benefits from the shorter duration indirectly, as she can extract more revenue from the bidders who trade off a shorter duration for a higher price. On aggregate, the social welfare is higher in the optimal Istanbul Flower Auction due to the time saved, and remains neutral with respect to the auction price, with the latter being a pure transfer from the bidders to the auctioneer.

Formally, we decompose the total effect of the change from the Dutch auction to the Istanbul Flower Auction on the auctioneer utility, bidder utility and the social welfare, into the time-saving (duration) and bidding-strategy (price) effects as follows:

$$\begin{aligned}\text{Total effect} &= EU(p^F, t^F) - EU(p^D, t^D)\\ &= \underbrace{EU(p^F, t^F) - EU(p^D, t^F)}_{\text{Bidding-strategy effect}} + \underbrace{EU(p^D, t^F) - EU(p^D, t^D)}_{\text{Time-saving effect}},\end{aligned}$$

where EU refers to the auctioneer utility, bidder utility or the social welfare to be decomposed, and p^F and t^F (correspondingly, p^D and t^D) denote the equilibrium price and duration under the optimal Istanbul Flower (correspondingly, Dutch) auction.

[10] We are grateful to the Associate Editor and an anonymous referee for suggesting this line of inquiry.

Table 2
Decomposition of effects: Istanbul Flower Auction vs Dutch.

Model	Auctioneer utility			Bidder utility			Social welfare		
	(p^F, t^F)	(p^D, t^F)	(p^D, t^D)	(p^F, t^F)	(p^D, t^F)	(p^D, t^D)	(p^F, t^F)	(p^D, t^F)	(p^D, t^D)
$(2, 0.1, \text{linear})$	0.334	0.316	0.316	0.326	0.345	0.308	0.661	0.661	0.624
Strategy effect	0.018 (5.7%)			-0.018 (-5.9%)			0.000 (0.0%)		
Time effect		0.000 (0.0%)			0.037 (12.0%)			0.037 (5.9%)	
Total effect			0.018 (5.7%)			0.019 (6.1%)			0.037 (5.9%)
$(2, 0.7, \text{linear})$	0.341	0.164	0.164	0.279	0.456	0.125	0.620	0.620	0.289
Strategy effect	0.177 (107.7%)			-0.177 (-141.0%)			0.000 (0.0%)		
Time effect		0.000 (0.0%)			0.331 (263.7%)			0.331 (114.2%)	
Total effect			0.177 (107.7%)			0.154 (122.7%)			0.331 (114.2%)
$(10, 0.1, \text{linear})$	0.814	0.803	0.803	0.090	0.101	0.088	0.904	0.904	0.892
Strategy effect	0.011 (1.4%)			-0.011 (-12.5%)			0.000 (0.0%)		
Time effect		0.000 (0.0%)			0.013 (14.2%)			0.013 (1.4%)	
Total effect			0.011 (1.4%)			0.001 (1.7%)			0.013 (1.4%)
$(10, 0.7, \text{linear})$	0.788	0.606	0.606	0.083	0.265	0.059	0.871	0.871	0.665
Strategy effect	0.182 (30.1%)			-0.182 (-308.1%)			0.000 (0.0%)		
Time effect		0.000 (0.0%)			0.206 (348.6%)			0.206 (31.0%)	
Total effect			0.182 (30.1%)			0.024 (40.4%)			0.206 (31.0%)

Notes: Percentage numbers are the relative scale of the effect as compared to the Dutch auction.

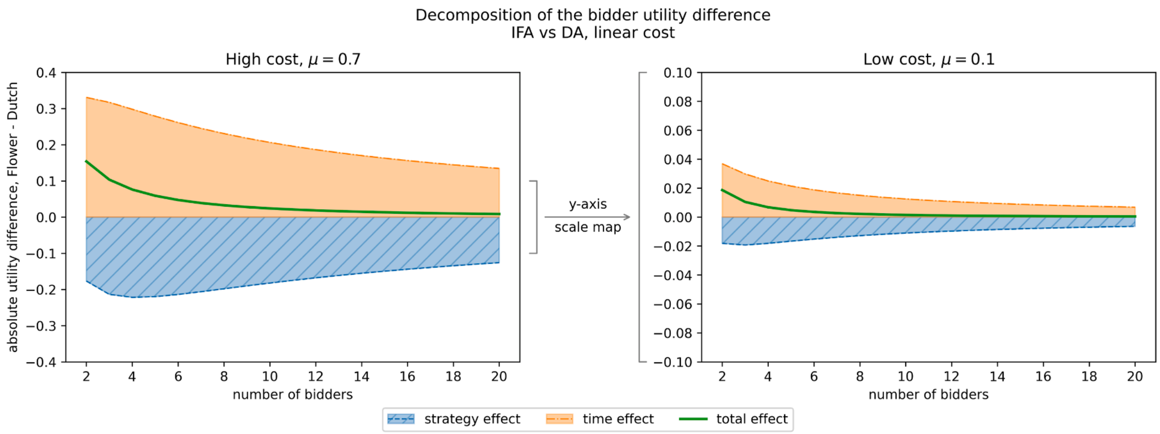


Fig. 4. Bidder strategy and time-saving effects of Istanbul Flower Auction over Dutch with a linear time cost $c(t) = 1 - \mu t$. *Notes*: The auctioneer's total effect is solely due to the strategy effect and is obtained by flipping the bidder strategy effect (denoted in blue) along the horizontal axis. The social planner's total effect is solely due to time-saving and exactly equals to the bidder time effect (denoted in yellow). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

To isolate *the time-saving effect*, we fix the price at p^D and vary auction durations between t^D and t^F. That is, we evaluate the difference in utilities between the Dutch auction and a counter-factual scenario of an auction that would result in the Dutch-auction price p^D and the optimal Istanbul Flower Auction duration t^F:

$$\text{Time-saving effect} = EU(p^D, t^F) - EU(p^D, t^D)$$

To isolate *the bidding-strategy effect*, we fix the auction duration at t^F and vary the auction prices between p^F and p^D. That is, we evaluate the difference in utilities between the optimal Istanbul Flower Auction and the aforementioned counterfactual scenario of an auction that ends at the Dutch price p^D but has the same duration t^F as the optimal Istanbul Flower Auction:

$$\text{Bidding-strategy effect} = EU(p^F, t^F) - EU(p^D, t^F)$$

By our model specifications (Section 3), only bidder values are discounted, whereas the price paid by the bidders and received by the auctioneer is not. Therefore, the time effect is always zero for the auctioneer, whose total effect is equal to the bidding-strategy effect. The bidding-strategy effect is positive for the auctioneer as long as she prefers the Istanbul Flower Auction to the Dutch auction. Consequently, the bidding-strategy effect is always negative for the bidders: since price is a pure transfer from the bidders to the auctioneer, the bidding-strategy effect of the bidders is the negative of the corresponding effect for the auctioneer. This implies that the auctioneer's and bidders' bidding strategy effects cancel out for the social welfare, which is unaffected by prices. From the social welfare perspective, the sole source of benefit of the Istanbul Flower Auction as compared to the Dutch auction comes from the time-savings effect for the bidders.

Illustrative example continued. Let us illustrate the two effects using the example from Section 3.5, with two bidders and the linear bidder time cost $c(t) = 1 - 0.5t$. The 48.7% improvement in the auctioneer utility from the Istanbul Flower Auction relative to the Dutch auction is solely due to the strategy effect, i.e., a higher expected auction price. The expected utility gain decomposition into the time and strategy effects for the bidders is performed by comparing their expected utility of 0.296 for the Istanbul Flower Auction, that of 0.193 for the Dutch auction, and the counter-factual expected utility of 0.407 for the auction with the Istanbul Flower Auction duration and the Dutch auction price. In relative terms, this implies a negative 57.3% change due to the strategy effect, and a positive 110.8% change due to the time-saving effect, with the total 53.5% improvement in bidder utility. Finally, the 50.9% improvement in the social welfare is solely due to the time-saving effect.

Table 2 and Fig. 4 illustrate the time-saving and bidding strategy effects for auctions with a different numbers of bidders, and high and low bidder cost of time. From these numerical results we confirm that the bidding strategy effect for the bidders is always negative, while the time-saving effect is positive. Yet the time-saving effect outweighs the utility loss due to a higher price paid to the auctioneer. Hence, adopting the Istanbul Flower Auction format is the win-win outcome from both the auctioneer's and the bidders' perspectives, resulting in higher utilities for all parties and a higher social welfare.

In sum, the Istanbul Flower Auction's main advantage lies in saving time in a time-costly environment: bidders are willing to pay higher prices to save time, with a time-saving benefit typically outweighing the payment sacrificed. Although the Istanbul Flower Auction mechanism is designed by the auctioneer to take advantage of bidder impatience, bidders, like the auctioneer, also prefer this mechanism over the slower Dutch auction.

6. Conclusion

Our analysis reveals that the Istanbul Flower Auction format, with its innovative approach to combining Dutch and English auction elements, offers distinct advantages in achieving higher utilities for both auctioneers and bidders. This format proves particularly beneficial in markets characterized by high time-sensitivity, such as perishable goods auctions. The flexibility to switch between different auction dynamics based on the initial bidding activity allows for more efficient price discovery and lower time costs, and can lead to higher utilities for all parties involved. Our numerical findings illustrate that both the auctioneer and the bidders prefer the Istanbul Flower Auction over the Dutch auction, and the Istanbul Flower Auction sells goods faster than the traditional Dutch auction. This insight aligns with real-world scenarios where goods need to be sold swiftly.

Our findings suggest that adopting flexible auction formats like the Istanbul Flower Auction could enhance outcomes for all the parties involved in various auction-based markets, particularly those where the speed of the auction is important due to the large number of items pending for sale in a fixed amount of time.

Data availability

No data was used for the research described in the article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Supplementary material

Supplementary material associated with this article can be found in the online version at 10.1016/j.geb.2026.02.013.

Appendix A. Proofs

Proof of Lemma 1. In contrast to the Dutch auction setting where the price continuously descends from $s = 1$ and one can solve for a strictly increasing bidding function, the bidding function here may be capped by the starting price s when the bidder's private value exceeds a certain cutoff $\lambda(s) \geq s$. Therefore, the bidding function for the subsequent Dutch auction is given by

$$\beta(v, s) = \begin{cases} b(v, s) & v \leq \lambda(s) \\ s & \lambda(s) \leq v \leq p(s) \end{cases}$$

and the bidder's expected utility is given by

$$EU_B^{FD}(v, s) = \begin{cases} [c(s - b(v, s))v - b(v, s)]G(v) & v \leq \lambda(s) \\ \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{v-s}{k+1} F^{n-1-k}(\lambda(s))[F(p(s)) - F(\lambda(s))]^k & \lambda(s) \leq v \leq p(s) \end{cases}$$

where $b(\cdot, s) \to [0, s]$ is a strictly increasing and differentiable function with $b(0) = 0$ given the starting price s.

Step 1. We prove $\lambda(s) = p(s)$ by contradiction in two cases, depending on whether the subsequent auction could have an English phase or not.

Let us start from the case $p(s) < 1$, where the auction could proceed with either Dutch or English phases. Suppose $\lambda(s) < p(s)$; then in equilibrium a bidder with value $v = p(s)$ should be indifferent between bidding or not at the starting price s. That is,

$$EU_B^{FD}(p(s), s) = EU_B^{FE}(p(s), s).$$

We know that

$$EU_B^{FE}(p(s), s) = (p(s) - s)G(p(s))$$

and

$$\begin{aligned} EU_B^{FD}(p(s), s) &= \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{p(s) - s}{k+1} F^{n-1-k}(\lambda(s))[F(p(s)) - F(\lambda(s))]^k \\ &= \frac{p(s) - s}{n} \frac{F^n(p(s)) - F^n(\lambda(s))}{F(p(s)) - F(\lambda(s))}. \end{aligned}$$

Let us define an auxiliary function

$$k(x) = \frac{1 - x^n}{n(1 - x)} = \frac{1 + x + \cdots + x^{n-1}}{n}.$$

It is obvious that $k'(x) > 0$ for $x \in (0,1)$, and $k(1) = 1$. Note that $\frac{F(\lambda(s))}{F(p(s))} \in (F(\lambda(s)), 1)$. Then we have

$$\frac{EU_B^{FD}(p(s),s)}{EU_B^{FE}(p(s),s)} = k\left(\frac{F(\lambda(s))}{F(p(s))}\right) < k(1) = 1.$$

which means we can never have the necessary condition for the equilibrium satisfied. Therefore, we conclude that there is no equilibrium such that $\lambda(s) < p(s)$ when $p(s) < 1$.

Now let us look into the case $p(s) = 1$, where the Istanbul Flower Auction becomes a Dutch auction with the starting price s. Suppose $\lambda(s) < p(s)$. Then, any Dutch bidder whose bid is capped at s will deviate to bidding at the starting price. Indeed she will win the item at the starting price for sure by turning the auction to the English phase when all other bidders wait for the Dutch phase.

Step 2. Given that $\lambda(s) = p(s)$ in equilibrium when $p(s) < 1$, we invoke the indifference condition at $v = p(s)$ again. We have

$$\begin{aligned} EU_B^{FD}(p(s),s) &= EU_B^{FE}(p(s),s) \\ [c(s - b(p(s),s))p(s) - b(p(s),s)]G(p(s)) &= (p(s) - s)G(p(s)) \\ c(s - b(p(s),s))p(s) - b(p(s),s) &= p(s) - s \end{aligned}$$

By the definition of the time cost function, we have $c(0) = 1$. Thus, $b(p(s),s) = s$ is a solution, and we denote this implicitly solved cutoff by p_0. Since we have proved in Step 1 that the Dutch phase bidding strategy is not capped (and therefore strictly increasing from 0 to at most s), for the uniqueness of the solved cutoff under a given s, we only need to confirm that no other $p \in [s, p_0)$ can satisfy the aforementioned indifference condition. This is equivalent to proving the uniqueness of the zero point in $[s, p_0]$ for the auxiliary function defined below:

$$l(p) = (p - s) - [c(s - b(p,s))p - b(p,s)].$$

It is obvious that $l(p)$ is differentiable and $l(p_0) = 0$. We compute its first-order derivative and reformat it by substituting in Eq. (1) evaluated at $v = p$. We have

$$\begin{aligned} l'(p) &= 1 - \left[-c'(s - b(p,s))\frac{\partial b(p,s)}{\partial p}p + c(s - b(p,s)) - \frac{\partial b(p,s)}{\partial p}\right] \\ &= 1 - c(s - b(p,s)) + \frac{g(p)}{G(p)}[c(s - b(p,s))p - b(p,s)]. \end{aligned}$$

For $p \in [s, p_0)$, by what we have proved in Step 1 for the Dutch phase bidding strategy, we have $s - b(p,s) > 0$. Then by the definition of the time cost function, we have $c(s - b(p,s)) < 1$. By individual rationality, we have $c(s - b(p,s))p - b(p,s) \geq 0$. Therefore, we have $l'(p) > 0$. Now we know that $l(p)$ is strictly increasing in (s, p_0) with $l(p_0) = 0$, which means it has a unique zero point p_0 in $[s, p_0]$.

When $p(s) = 1$, we do not necessarily have $b(p,s) = s$ since the starting price s may be too high to put any restriction on any bidder's Dutch phase decision. However, it must be true that $b(p,s) \leq s$. □

Proof of Lemma 2. Taking the partial derivative with respect to s on both sides of Eq. (1), we have

$$\frac{\partial^2 b(v,s)}{\partial s \partial v} = \frac{g(v)}{G(v)} \frac{\left[c'(s - b(v,s))\left(1 - \frac{\partial b(v,s)}{\partial s}\right)v - \frac{\partial b(v,s)}{\partial s}\right][1 + c'(s - b(v,s))v] - [c(s - b(v,s))v - b(v,s)]c''(s - b(v,s))\left(1 - \frac{\partial b(v,s)}{\partial s}\right)v}{[1 + c'(s - b(v,s))v]^2}.$$

The above equation can be viewed as a differential equation of the function $\frac{\partial b(v,s)}{\partial v}$ and its derivative with respect to s. Since we focus on the well-behaved increasing bidding function $b(v,s)$, that allows us to interchange the order of partial derivatives such that $\frac{\partial^2 b(v,s)}{\partial s \partial v} = \frac{\partial^2 b(v,s)}{\partial v \partial s}$. Note that We also have the initial condition $b(0) \equiv 0$, which gives the initial condition $\frac{\partial b(v,s)}{\partial s}|_{v=0} = 0$ for the above differential equation. Following the path of $\frac{\partial b(v,s)}{\partial s}$, when the value of the function $\frac{\partial b(v,s)}{\partial s} \to 1$ at certain v, its derivative $\frac{\partial}{\partial v}\frac{\partial b(v,s)}{\partial s} < 0$ at that point. Therefore, starting from the value of $\frac{\partial b(0,s)}{\partial s} \equiv 0$, the function $\frac{\partial b(v,s)}{\partial s}$ is always below the upper bound that $\frac{\partial b(v,s)}{\partial s} < 1$. □

Proof of Lemma 3. By definition, we have

$$c(m(v,s) - s)v - m(v,s) = 0.$$

When there is no time cost, $c(t) = 1$, we have the classical result $m(v,s) = v$.

Otherwise, we must have $c'(t) < 0$. Taking the derivative with respect to s on both sides, we have

$$c'(m(v,s) - s)\left(\frac{\partial m(v,s)}{\partial s} - 1\right)v - \frac{\partial m(v,s)}{\partial s} = 0$$

which gives

$$\frac{\partial m(v,s)}{\partial s} = -\frac{c'(m(v,s) - s)v}{1 - c'(m(v,s) - s)v} \in (0,1)$$

□

Proof of Lemma 4. The starting price s puts a cap on the Dutch phase bids. By Lemma 2, we know that the Dutch phase bidding function will never move upward faster than the cap s. It is also evident that $b(1, s) < 1$. Therefore, when s increases from 0 to 1, it departs upwards from the Dutch phase bidding function and finally becomes non-restrictive to the Dutch phase bidding strategy after reaching some interim $\tilde{s}$. By Lemma 1, we can solve for $\tilde{s}$ from $b(1, \tilde{s}) = \tilde{s}$, and use the monotonicity of the $l(\cdot)$ function defined in the proof of that lemma to prove that $p(s) \equiv 1$ for all $s \in [\tilde{s}, 1]$.

Now, let us consider $s \in [0, \tilde{s})$ where the Dutch phase bids of high-value bidders are capped. By Lemma 1 we know that

$$b(p(s), s) = s.$$

Taking the derivative with respect to s on both sides of the above equation, we have

$$\left.\frac{\partial b(v,s)}{\partial v}\right|_{v=p(s)} \frac{dp(s)}{ds} + \left.\frac{\partial b(v,s)}{\partial s}\right|_{v=p(s)} = 1.$$

We are considering strictly increasing Dutch bidding strategies, which means $\frac{\partial b(v,s)}{\partial v} > 0$. By Lemma 2, we have $\frac{\partial b(v,s)}{\partial s} < 1$. Therefore, we conclude that

$$\frac{dp(s)}{ds} > 0.$$

□

Proof of Lemma 5. From the initial condition of the Dutch phase differential Eq. (1), $b(0, s) = 0$ as the bidder with zero value always bids zero, we obtain that $\frac{\partial b(0,s)}{\partial s} = 0$.

Suppose that there exists $s_1 < s_2, v_0 \in (0, 1]$ such that $b(v_0, s_1) = b(v_0, s_2) = b_0$, from the Dutch phase differential Eq. (1) we have

$$\frac{\partial b(v_0, s_1)}{\partial v} \frac{1 + v_0 c'(s_1 - b_0)}{v_0 c(s_1 - b_0) - b_0} = \frac{\partial b(v_0, s_2)}{\partial v} \frac{1 + v_0 c'(s_2 - b_0)}{v_0 c(s_2 - b_0) - b_0}.$$

Under the assumption, we know that $c'(s_1 - b_0) \le c'(s_2 - b_0) < 0$, while the equal sign only holds for the linear cost. We also have, by definition of the cost function, $c(s_1 - b_0) > c(s_2 - b_0) > 0$. Then we must have

$$\frac{\partial b(v_0, s_1)}{\partial v} > \frac{\partial b(v_0, s_2)}{\partial v}.$$

For $h \to 0$, applying Euler's method to the differential equation with initial condition $b(0, s) = 0$ to get

$$b(h, s) \approx b(0, s) + h\frac{\partial b(0, s)}{\partial v} = 0,$$

and substitute this into the Dutch differential equation to get

$$\frac{\partial b(h, s)}{\partial v} \approx \frac{g(h)}{G(h)} \frac{hc(s)}{1 + hc'(s)}.$$

Again, given that $s_1 < s_2$, we must have $c(s_1) > c(s_2) > 0$ by definition of the cost function, and under the assumption we have $c'(s_1) \le c'(s_2) < 0$. Then we have

$$\frac{\partial b(0, s_1)}{\partial v} > \frac{\partial b(0, s_2)}{\partial v}.$$

Now we know that, for any starting price $s_1 < s_2$, the two Dutch phase bidding functions $b(v, s_1)$ and $b(v, s_2)$ have the following properties: (i) the two functions share the same starting point $b(0, s) = 0$; (ii) $b(v, s_1)$ is steeper than $b(v, s_2)$ at $v = 0$; (iii) whenever $b(v, s_1)$ and $b(v, s_2)$ have a non-zero intersection point, $b(v, s_1)$ is steeper than $b(v, s_2)$ at that point. Suppose such non-zero intersection points exist, we must be able to find the smallest among them, denoted by $v_m > 0$. By the aforementioned properties, there exists $\varepsilon_1, \varepsilon_2 > 0$ such that $b(v_m - \varepsilon_1, s_2) > b(v_m - \varepsilon_1, s_1), b(\varepsilon_2, s_2) < b(\varepsilon_2, s_1)$. Then, according to the intermediate value theorem, there must be another intersection point between 0 and v_m, which gives a contradiction. Therefore, we conclude that

$$\frac{\partial b(v, s)}{\partial s} < 0, \forall v \in (0, 1].$$

□

Proof of Proposition 2. By Lemmas 1 and 4, we have either

$$s \in [0, \tilde{s}), p(s) < 1, \frac{dp(s)}{ds} > 0, b(p(s), s) = s$$

or

$$s \in (\tilde{s}, 1], p(s) = 1, \frac{dp(s)}{ds} = 0, b(p(s), s) < s.$$

Specifically, $p(s)$ is not differentiable at $\tilde{s}$: the left-sided limit belongs to the first case, and the right-sided limit belongs to the second case. However, plugging these into Eq. (7) indicates that $EU_A^F(s)$ is differentiable at that point, and the first-order derivative of the auctioneer utility for $s \in [\tilde{s}, 1)$ is given by

$$\frac{dEU_A^F(s)}{ds} = \int_0^1 \frac{\partial b(v, s)}{\partial s} dF^n(v) < 0$$

because by Lemma 5 we have $\frac{\partial b(v,s)}{\partial s} < 0$ for all $v \in (0,1]$. Therefore, we must have $EU_A^F(s^*) > EU_A^F(s) \geq EU_A^F(1) = EU_A^D$ for any $s \in [\tilde{s}, 1]$. It is obvious that $s^* < \tilde{s}$.

Using Eq. (7) again, we can compute that

$$\frac{dEU_A^F(0)}{ds} = -\int_0^1 \int_0^v \frac{\partial m(v,s)}{\partial s} h(v,x)\, dx\, dv < 0$$

because by Lemma 3 we have $\frac{\partial m(v,s)}{\partial s} \in (0,1)$. Therefore, we must have $EU_A^F(s^*) > EU_A^F(0) = EU_A^D$ for any $s \in [\tilde{s}, 1]$, which also implies $s^* > 0$.

Finally, we can conclude that $s^* \in (0, \tilde{s})$, and therefore the optimal auction is non-trivial. □

Proof of Proposition 3. Below we show that the first-order derivative of the expected utility for bidders is (1) increasing at $s = 0$ so there exists a better Istanbul Flower Auction than the standard English auction, and (2) decreasing in $(\tilde{s}, 1)$ so there exists a better Istanbul Flower Auction than the standard Dutch auction. Given the obvious continuity of the utility function, the proposition then follows naturally.

(1) The first-order derivative of the bidder utility at $s = 0$ is given by

$$\frac{dEU_B^F(0)}{ds} = \int_0^1 \int_0^v \left[c'(m(x,0))\left(\frac{dm(x,0)}{ds} - 1\right)v - \frac{dm(x,0)}{ds}\right] dG(x)\, dF(v).$$

Taking the derivative with respect to s on both sides of Eq. (2) we have

$$c'(m(v,s))\left(\frac{dm(v,s)}{ds} - 1\right)v - \frac{dm(v,s)}{ds} = 0$$

which implies

$$c'(m(x,s))\left(\frac{dm(x,s)}{ds} - 1\right)v - \frac{dm(x,s)}{ds} \geq 0$$

where the equality holds only for $x = v$, because by Assumption 1 we have $c'(t) < 0$, by Lemma 3 we have $\frac{dm(x,s)}{ds} < 1$, and by our model setting x is the second highest value which must be less than or equal to v. Therefore, considering the integrand of the derivative, we have that the strict inequality holds, that is $\frac{dEU_B^F(0)}{ds} > 0$.

(2) The first-order derivative of the bidder utility for $s \in [\tilde{s}, 1)$ is given by

$$\frac{dEU_B^F(s)}{ds} = \int_0^1 \left(c'(s - b(v,s))v - [1 + c'(s - b(v,s))v]\frac{\partial b(v,s)}{\partial s}\right) G(v)\, dF(v)$$

Let us denote the integrand by

$$w(v,s) = G(v)\left(c'(s - b(v,s))v - [1 + c'(s - b(v,s))v]\frac{\partial b(v,s)}{\partial s}\right)$$

By the initial condition $b(0) = 0$ for the Dutch phase differential equation, we know that $\frac{\partial b(0,s)}{\partial s} = 0$ and then we have $w(0,s) = 0$. Taking the derivative with respect to v, we have

$$\begin{aligned} \frac{\partial w(v,s)}{\partial v} = {} & G(v)\Big(\big[-c''(s - b(v,s))\frac{\partial b(v,s)}{\partial v}v + c'(s - b(v,s))\big]\Big(1 - \frac{\partial b(v,s)}{\partial s}\Big) \\ & - \big[1 + c'(s - b(v,s))\, v\big]\frac{\partial^2 b(v,s)}{\partial s\, \partial v}\Big) \\ & + g(v)\Big(c'(s - b(v,s))\, v - \big[1 + c'(s - b(v,s))\, v\big]\frac{\partial b(v,s)}{\partial s}\Big). \end{aligned} \tag{A.1}$$

Substitute in $\frac{\partial^2 b(v,s)}{\partial v \partial s}$,

$$\begin{aligned} \frac{\partial^2 b(v,s)}{\partial v \partial s} = {} & \frac{g(v)}{G(v)} \frac{\left[c'(s - b(v,s))\left(1 - \frac{\partial b(v,s)}{\partial s}\right)v - \frac{\partial b(v,s)}{\partial s}\right][1 + c'(s - b(v,s))v]}{[1 + c'(s - b(v,s))v]^2} \\ & - \frac{g(v)}{G(v)} \frac{[c(s - b(v,s))v - b(v,s)]c''(s - b(v,s))\left(1 - \frac{\partial b(v,s)}{\partial s}\right)v}{[1 + c'(s - b(v,s))v]^2} \end{aligned}$$

we have $\frac{\partial w(v,s)}{\partial v}$ simplified as:

$$\begin{aligned} \frac{\partial w(v,s)}{\partial v} = {} & G(v)\left[-c''(s - b(v,s))\frac{\partial b(v,s)}{\partial v}v + c'(s - b(v,s))\right]\left(1 - \frac{\partial b(v,s)}{\partial s}\right) \\ & + g(v)\frac{(c(s - b(v))v - b)c''(s - b(v,s))(1 - \frac{\partial b(v,s)}{\partial s})v}{1 + c'(s - b(v,s))v} \end{aligned}$$

Substitute in $\frac{\partial b(v,s)}{\partial v}$ in (1):

$$\frac{\partial w(v,s)}{\partial v} = G(v)c'(s-b(v,s))\left(1-\frac{\partial b(v,s)}{\partial s}\right)$$

By Assumption 1 and Lemma 2, we know that $c'(t)<0$ and $1-\frac{\partial b(v,s)}{\partial s}>0$. Therefore, $\forall v\in(0,1)$, $w(v,s)$ is decreasing in v with $s\in(\tilde{s},1)$. Combining with $w(0,s)\equiv 0$ as the bidder with zero value always gets zero utility, we have $\frac{dEU_B^F(s)}{ds}<0$ for $s\in(\tilde{s},1)$.

□

Proof of Proposition 4. Differentiating Eq. (5) with respect to the starting price s, for $s\in(\tilde{s},1)$, we have

$$\frac{\partial EU_S^F(s)}{\partial s} = \int_0^1 c'(s-b(v,s))\left(1-\frac{\partial b(v,s)}{\partial s}\right)v\,dF^n(v)<0$$

and for $s=0$, we have

$$\frac{\partial EU_S^F(s)}{\partial s} = \int_0^1\int_0^v c'(m(v,s)-s)\left(\frac{\partial m(v,s)}{\partial s}-1\right)vh(v,x)\,dx\,dv>0$$

since both $\frac{\partial b(v,s)}{\partial s}<1$ and $\frac{\partial m(v,s)}{\partial s}<1$. The proposition directly follows. □

Proof of Proposition 5. Differentiating Eq. (6) with respect to the starting price s, for $s\in(\tilde{s},1)$, we have

$$\frac{\partial ED^F(s)}{\partial s} = \int_0^1\left(1-\frac{\partial b(v,s)}{\partial s}\right)dF^n(v)>0$$

and for $s=0$, we have

$$\frac{\partial ED^F(s)}{\partial s} = \int_0^1\int_0^v\left(\frac{\partial m(v,s)}{\partial s}-1\right)h(v,x)\,dx\,dv<0$$

since both $\frac{\partial b(v,s)}{\partial s}<1$ and $\frac{\partial m(v,s)}{\partial s}<1$. The proposition directly follows. □

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